



॥ विद्या सर्वस्य भूषणम् ॥

PRABODHAN EDUCATION SOCIETY'S

Vidya Prabodhini College of Commerce, Education, Computer & Management
Vidyanagar, Alto-Parvari, Goa

SEMESTER END EXAMINATION – OCTOBER 2016 (Regular/Repeat)

Sub: MATHEMATICAL TECHNIQUES - I

Semester: I

M. Marks: 80

M. Time: 2 hrs.

INSTRUCTIONS: 1. All questions are compulsory however internal choice is available.

2. Use of calculator is strictly forbidden.

3. Figures to the right indicate marks allotted.

4. There are 4 main questions each carrying 20 marks.

5. You may answer randomly but every main question attempted should be answered serially.

Q1. Attempt the following.

(4 x 5 = 20)

- Construct the truth table for $[\neg p \wedge (q \leftrightarrow p)] \rightarrow q$. What can you conclude from the last column of the truth table?
- If $9({}^{n-1}P_3) = {}^nP_4$, then find n .
- If the 5th term of an A.P is 21 and its 8th term is 39, find its n th term.
- Find the matrix X such that $3X + \begin{bmatrix} 3 & 4 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 1 & 1 \end{bmatrix}$.

OR

Q1. Attempt the following.

(4 x 5 = 20)

- Examine whether the statement $\neg(p \wedge q) \rightarrow [\neg p \vee (\neg p \vee q)]$ is tautology, contradiction or neither.
- A council consist of 10 members, 6 belonging to party A and 4 belong to party B. In how many ways can a committee of 5 be formed so that the members of the party B are in majority?
- Find the sum $4 + 7 + 10 + 13 + \dots + 49$.

- If $A = \begin{bmatrix} 2 & 0 \\ 3 & 7 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 1 \\ -1 & 2 \\ 2 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 0 \\ 4 & -2 \\ -3 & -2 \end{bmatrix}$ then find a matrix X such that $2A + 3B + X = 2C$.

Q2. Attempt the following.

(4 x 5 = 20)

- In a survey of 100 families, 42 read India Today and 48 read Readers' Digest while 13 read both the magazines. How many read neither India Today nor Readers' Digest?

b) A committee of 4 is to be selected from 5 boys and 6 girls. In how many ways can this be done so that at least one boy is in the committee?

c) Find x , if
$$\begin{vmatrix} x & 2 & 1 \\ 3 & 0 & 1 \\ 4 & -5 & 2 \end{vmatrix} = 0.$$

d) A person pays ₹ 975 in monthly installments such that each installment is less than the previous installment by ₹ 5. If the first installment is ₹ 100, in how many months will the entire amount be paid?

OR

Q11. Attempt the following.

(4 x 5 = 20)

p) If $A = \{2, 4, 7, 9\}$, $B = \{1, 5, 7\}$ and $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is the universal set, verify that

i) $B^c - A^c = A - B$

ii) $B - A = B \cap A^c$

q) Find n , if ${}^{2n}P_1 = 36({}^nP_2)$.

r) Without expanding the following determinant prove that
$$\begin{vmatrix} x-y & x+y & x \\ z-x & z+x & z \\ y-z & y+z & y \end{vmatrix} = 0.$$

s) Shital is paying her debt in 8 monthly installment which form a Geometric Progression. If her third installment is ₹ 100 and the sixth installment is ₹ 12500 then find her first installment and the entire amount repaid by her.

Q3. Attempt the following.

(4 x 5 = 20)

a) Check whether the statements $\neg(p \leftrightarrow q)$ and $(p \wedge \neg q) \vee (\neg p \wedge q)$ are equivalent.

b) In a hostel, 15 members take tea, 8 members take coffee and 6 members take milk. If 5 members take tea and coffee both, 4 members take tea and milk both and if none of them take coffee and milk both or all three, then find the number of members in the hostel. (Every member takes at least one of the three beverages)

c) If $A = \begin{bmatrix} 2 & 1 \\ -5 & 3 \end{bmatrix}$, find $A^2 - 5A + 3I$, where I is the identity matrix of order 2.

d) Find 5th term in the expansion of $\left[2x^2 - \frac{3}{2x}\right]^7$.

OR

Q11. Attempt the following.

(4 x 5 = 20)

p) State whether the statement $(p \rightarrow q) \rightarrow [(q \rightarrow r) \rightarrow (p \rightarrow r)]$ is tautology or not.

q) Let $X = \{1, 2, 3, \dots, 9, 10\}$ be universal set. If $A = \{1, 2, 3, 4, 5, 7\}$, $B = \{2, 4, 5, 6, 7, 8\}$ and $C = \{4, 6, 8, 9, 10\}$ then verify that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

r) If $A = \begin{bmatrix} 1 & 0 \\ 5 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -2 \\ 1 & 3 \end{bmatrix}$ then verify that $(AB)^T = B^T \cdot A^T$

s) Find the term independent of x in $\left[x^2 - \frac{1}{x}\right]^9$.

Q4. Attempt the following.

(4 x 5 = 20)

a) How many permutations can be made with the letters of the word FORMULA? In how many of them are M and A together?

b) By using binomial theorem prove that $(\sqrt{2} + 1)^5 - (\sqrt{2} - 1)^5 = 82$

c) Solve the system of equations using Cramer's rule

$$\begin{aligned} 2x - 3y &= 5, \\ 3x + 4y + 1 &= 0 \end{aligned}$$

d) Find the inverse of the matrix $A = \begin{bmatrix} 2 & -1 \\ 4 & 2 \end{bmatrix}$.

OR

QIV. Attempt the following.

(4 x 5 = 20)

p) Find the value of ${}^{10}C_6 + {}^{10}C_5 - {}^9C_6 - {}^9C_5$.

q) Find the coefficient of x^9 in $\left(\frac{1}{x^2} + x\right)^{18}$

r) If Find the value of x and y from the matrix equation

$$\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x & 5 & 3 \\ 2 & y & -5 \end{bmatrix} = \begin{bmatrix} 5 & -3 & -7 \\ 7 & 7 & -1 \end{bmatrix}$$

s) Evaluate $\begin{vmatrix} x+y & x-y \\ x-y & x+y \end{vmatrix}$