



॥ विद्या सर्वस्य भूषणम् ॥

PRABODHAN EDUCATION SOCIETY'S

Vidya Prabodhini College of Commerce, Education, Computer & Management  
Vidyanagar, Alto-Parvari, Goa

**SEMESTER END EXAMINATION – APRIL 2017 (Regular/Repeat)**

**Sub: MATHEMATICAL TECHNIQUES - II**

**Semester: II**

**M. Marks: 80**

**M. Time: 2 hrs.**

INSTRUCTIONS: 1. All questions are compulsory however internal choice is available.

2. Use of calculator is strictly forbidden.

3. Figures to the right indicate marks allotted.

4. There are 4 main questions each carrying 20 marks.

5. You may answer randomly but every main question attempted should be answered serially.

**Q 1) Attempt the following.**

(4 x 5 = 20)

a) Prove that the points (-3, 4), (3, 10) and (6, 7) are the vertices of a right angled triangle.

b) Find  $\frac{dy}{dx}$  if (i)  $y = 3\sqrt{x} - x^4 \log(x)$  (ii)  $y = \frac{(4x-3)}{(x^2+1)}$

c) Find the domain and range of the function  $f(x) = 7x - 3$ ,  $-2 \leq x \leq 3$ .  
Also find  $f(-1)$ ,  $f(2)$  and  $f(4)$  if exist.

d) A man intends to invest a sum of money which will amount to ₹ 5000 in 10 years at 4% per annum compound interest. What amount should he invest now?

OR

**Q 1) Attempt the following.**

(4 x 5 = 20)

p) Find the lengths of the sides of triangle ABC given that the vertices are  $A \equiv (4, -5)$ ,  
 $B \equiv (3, 2)$  and  $C \equiv (-3, 2)$ . Is the triangle an isosceles triangle? Give reason.

q) Find  $\frac{dy}{dx}$  if (i)  $y = (4x-3)^5$  (ii)  $y = \frac{3x^2-4}{\sqrt{x}}$

r) If  $f(x) = \frac{x+3}{4x-5}$  and  $g(x) = x^2 + 2$ , find  $f(g(x))$ . Also find  $f[g(1)]$ ,  $f[g(-2)]$ .

s) In how many years will ₹5300 give ₹1113 as simple interest at 7% per annum?

**Q 2) Attempt the following.**

(4 x 5 = 20)

a) Find i)  $\int \frac{3x+5}{3x-2} dx$ , ii)  $\int \left[ (2x-5)^3 + \frac{1}{(9-2x)^3} \right] dx$

b) If  $z = x^2 + 2xy - y^2$ , find  $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2}$ .

c) Find  $\lim_{x \rightarrow 4} \left( \frac{\sqrt{5+x} - 3}{x^2 - 3x - 4} \right)$

- d) Find the equation of the straight line passing through the point (1, -3) and the mid-point of segment PQ where  $P \equiv (3, -4)$ ,  $Q \equiv (-5, 8)$ .

OR

Q II) Attempt the following.

(4 x 5 = 20)

- p) Integrate the following w.r.t. x (i)  $\frac{1}{4x-3} + \frac{1}{(4x-3)^2}$  (ii)  $(1-x)x^3$

q) If  $z = x^3 + 3xy + y^3$ , then find  $x^2 \frac{\partial^2 z}{\partial x^2} - 4xy \frac{\partial^2 z}{\partial x \partial y}$ .

r) A function f is defined as  $f(x) = \begin{cases} \frac{x^2 - 3x + 2}{x^2 + 2x - 3} & , x \neq 1 \\ -1/4 & , x = 1 \end{cases}$

Examine the continuity of f(x) at  $x = 1$ .

- s) Find the equation of line passing through the point (1, -5) and perpendicular to the line passing through the points (4, 3) & (-5, 2).

Q 3) Attempt the following.

(4 x 5 = 20)

- a) Solve the following LPP by graphical method.

$$\text{Minimize } z = 12x + 18y, \quad \text{subject to}$$

$$x + 2y \geq 30,$$

$$3x + y \geq 30, \quad x \geq 0, \quad y \geq 0.$$

- b) If  $f(x) = 15 + 12x - 3x^2$ , find the value of x for which f(x) is maximum.

- c) The supply function for a commodity is  $p = 3x^2 + 5$ . Find the producer's surplus at  $x = 3$ .

- d) Find the principle amount to be invested at 8% per annum so that after 4 years the amount will be ₹10,000 when the interest is compounded annually?

OR

Q III) Attempt the following.

(4 x 5 = 20)

- p) Solve the following LPP by graphical method

$$\text{Maximize } z = 5x + 8y, \quad \text{subject to}$$

$$5x + y \leq 27,$$

$$3x + 2y \leq 48, \quad x \geq 0, \quad y \geq 0.$$

- q) The total cost function is  $C = x^3 - 100x^2 + 15x$ , where x is the number of units produced. Find the number of units for which the average cost is minimum.

- r) The demand and supply functions for a commodity are  $p = 24 - 4x$  and  $p = 16x + 4$ . Find the consumer's surplus at the equilibrium price.

- s) Find the sum borrowed by Rohit from a bank on compound interest of 5% per annum, to be calculated annually, if he had to pay back ₹26,460 after 2 years.

**Q 4) Attempt the following.**

**(4 x 5 = 20)**

- a) If  $A \equiv (2, b)$ ,  $B \equiv (5, a)$ ,  $C \equiv (3, -3)$  and  $D \equiv (-2, 1)$  are vertices of parallelogram, then find value of  $a$  and  $b$  using slopes.
- b) Find  $\lim_{x \rightarrow 2} \left( \frac{x^{15} - 2^{15}}{x^9 - 2^9} \right)$ .
- c) Find  $\frac{d^2 y}{dx^2}$  where  $y = 4x^3 - 7x^2 + 3e^x$
- d) Find the value of  $\int_{-4}^2 (3x^2 - 6) dx$

**OR**

**Q IV) Attempt the following.**

**(4 x 5 = 20)**

- p) If  $A \equiv (2, -3)$  and  $B \equiv (-6, 8)$  are points then find the equation of line having  $x$  - intercept 5 and parallel to the line AB.
- q) Discuss the continuity of function at  $x = 2$  if  $f(x) = \begin{cases} \frac{3x^2 - 7}{x} & , 0 \leq x \leq 2 \\ \frac{x^3 - 2x - 4}{x^2 - 4} & , 2 < x \leq 5 \end{cases}$
- r) The total cost function is given by  $C = x^2 + 3x + 24$ . Find average cost, marginal cost at  $x = 10$  and  $x = 15$ .
- s) If  $\int_1^2 (2x^2 + 5x + c) dx = 16$ , find  $c$ .